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TECHNICAL TRANSLATION

F-24

THE COMPLEX CONDUCTIVITY OF PLASMA OF AN ARC DISCHARGE
SUPPORTED BY A DIRECT CURRENT

By Jiri Kracik

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THE COMPLEX CONDUCTIVITY OF PLASMA OF AN ARC DISCHARGE SUPPORTED BY A DIRECT CURRENT*

By Jiri Kracik**

An expression is sought for the complex conductivity of the plasma of an arc discharge supported by a strong d-c current for the case in which a very weak high-frequency current that does not affect the condition of the plasma passes through it. The agreement between the results found and previous notions is satisfactory.

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1. INTRODUCTION

Hitherto we have not had a clear idea of how great is the impedance of a high-frequency signal in a discharge when that signal is superimposed on a relatively strong d-c current that is supporting that discharge. Unlike high-frequency discharges in which the energy balance is determined by the energy of the high-frequency electric field, in our case the signal's energy is quite negligible in comparison to the energy supporting the d-c electric field.

Eccles /6/ has derived a complex expression for the dielectric constant of a plasma, from which a complex relationship can be obtained for the specific conductivity of a high-frequency signal in this plasma. A similar problem was the object of detailed work by Margenau /4/, and later by Everhart and Brown /5/. However, all the experiments were concerned actually with the high-frequency discharge, i.e., the plasma was maintained by high-frequency energy.

Cobine, Cleary, and Gray /3/ first measured the high-frequency impedance of discharge according to the system used in this article. The expressions derived by Margenau were not applicable in this case.

Margenau's basic assumption is that the distributive function of the electron velocities in the discharge plasma is determined by the intensity of the high-frequency electric field. However in /3/ the decisive factor in the distributive function of the electron velocities is apparently the d-c electric field.

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It was thus necessary to find a theoretical solution for the problem again. It was also necessary to make at least an approximate experimental investigation of how the imaginary component of conductivity changes with changes in the supporting current and how it changes with changes in frequency. The criterion for the new theory was apparently the fact that some of the known expressions must be applicable to the specific conductivity of d-c current.

It may be expected that the particular phenomenon must occur in any type of discharge, provided of course the basic requirement is met, i.e., the high-frequency signal must be negligible in comparison to the signal supporting the discharge. This condition is best fulfilled in arc discharge.

Boltzmann's kinetic equation must be used to calculate complex conductivity. Of course, this equation is practically insoluble for arc discharge, because of the complexity of phenomena occurring here. But it was not our intention to solve Boltzmann's equation for arc discharge, but rather to show that at least an approximate solution of this equation, with simplifications, yields results for the complex conductivity of a weak high-frequency signal that can be considered as approximately accurate.

2. SOLUTION OF BOLTZMANN'S KINETIC EQUATION

CALCULATION OF SPECIFIC CONDUCTIVITIES

It is well known /1/ that Boltzmann's equation, ignoring the magnetic field, ignoring changes in the distributive function resulting from ionization and recombination of electrons, and assuming that the plasma is spatially homogeneous, has the following form for electrons:

$$\frac{\partial}{\partial t} f_e + \frac{e_0}{m_e} \bar{E} \, \mathrm{grad}_c f_e + \frac{1}{n_e}(a - b) = 0 \qquad (1)$$

where f_e is the complete distributive function for electrons, e_o is the charge on the electron, m_e is its mass, $\bar{E}$ is the resultant intensity of the electric field causing the directed movement of electrons, and $\mathrm{grad}_c f_e$ is the change in the complete distributive function in the velocity space, the term (a-b) represents a change in the distributive function resulting from elastic and inelastic collisions, and n_e is the concentration of electrons.

It is understandable that, because of their small mass, the movement of electrons provides almost all the transfer of electric charges through the plasma; i.e., the discharge stream is represented by the movement of electrons. For this reason it is important to know the distributive function of electrons. In a stabilized state the increase in electrons per unit volume per unit time within a chosen velocity range approximately equals their loss through recombination. In arc discharge the concentration of electrons along the positive column can be considered constant, so that it is quite justified to ignore the term representing spatial nonhomogeneity of the arc plasma within the framework of the foregoing concepts. Therefore, although we do not consider the axial symmetry of the positive column, all the results are in the nature of average values with respect to the cross section of the arc body.

We shall take the intensity of the electric field as the intensity resulting from the application of a voltage to the electrodes, i.e., the superposition of a d-c electric field, E_{d-c}, and a high-frequency field, E_{hf} is

$$E = E_{d-c} + E_{hf} \cdot \cos \omega t \tag{2}$$

where ω is the angular frequency of the high-frequency signal and E_{hf} is, in this case, its amplitude.

According to /1/ and /10/, as we know, Lorentz' limited development can be used for f_e. It must be chosen so as to express the decisive fact for our case, i.e., that E_{d-c} determines the state of the plasma. Thus let us assume that the kinetic equation (1) is solved in the following form:

$$f_e = f_0 + \frac{v_x}{c} f_1 + \frac{v_x}{c} g_1 \cdot \cos \omega t + \frac{v_x}{c} g_2 \cdot \sin \omega t \tag{3}$$

where f_o, f_1, g_1, and g_2 are functions of the velocity of the electron $\underline{c}$ and v_x is the velocity of the electron along the $\underline{x}$-axis of the coordinate system chosen. Here f_o is the basic distributive function, f_1 is the distributive function of electron velocities, corresponding apparently to E_{d-c} and g_1, and g_2 is the distributive function corresponding to E_{hf}. It is clear from Eq. (3) that, in contrast to /1/ and /10/, here we are dealing with superposition of both electric fields. Whereas it is assumed that $E_{hf} \ll E_{d-c}$ (the condition of the operation), it will still be true that not only is $f_1 \ll f_o$, but $g_1 \ll f_1$, and $g_2 \ll f_1$. Then E_{hf} will have no effect whatever on the number of electron collisions or on the state of the plasma, which will actually be determined only by the functions f_o and f_1; and the high-frequency signal in its passage through the plasma will merely stabilize the state of the plasma (of the positive column). If we consider that the number of elastic electron collisions will be much greater than the number of inelastic (exciting) collisions then, in view of the fact that we cannot ignore the temperature of the neutral molecules, T_g, we can use Davydov's expression /11, 12, 13, 1/ instead of /14/ for the term (a-b), i.e.,

$$(a - b) = - \frac{m_e}{m_g} n_e \frac{1}{c^2} \frac{\partial}{\partial c}\left(\frac{c^3 k T_g}{m_e \cdot \lambda} \frac{\partial f_0}{\partial c} + \frac{c^4}{\lambda} f_0\right) + \frac{n_e}{\lambda} v_x f_1 \tag{4}$$

if m_g is the mass of neutral molecules or atoms in the plasma, $\underline{k}$ is Boltzmann's constant, and λ is the mean free path of the electron. Obviously, $\left[\partial(v_x/c)/\partial t\right] = 0$, $\left[\partial f_0/\partial t\right] = 0$, and $\left[\partial(v_x f_1/c)/\partial t\right] = 0$, but $\left[\partial\left(v_x g_1 \cos \omega t/c\right)/\partial t\right] = -(v_x/c) g_1 \sin \omega t$ and $\left[\partial\left(v_x g_2 \sin \omega t/c\right)/\partial t\right] = \left(v_x/c\right) g_2 \omega \cos \omega t$. Accordingly, Eqs. (2) through (4), Eq. (1) changes to:

$$\omega(-g_1 \sin \omega t + g_2 \cos \omega t)\frac{v_x}{c} + \frac{e_0}{m_e}\left(E_{d-c} + E_{hf} \cdot \cos \omega t\right) \cdot \mathbf{grad}_c\left(f_0 + \frac{v_x}{c} f_1 + \frac{v_x}{c} g_1 \cdot \cos \omega t + \frac{v_x}{c} g_2 \cdot \sin \omega t\right) - \frac{m_e}{m_g} \frac{1}{c^2} \frac{\partial}{\partial c}\left(\frac{c^3 k T_g}{m_e \lambda} \frac{\partial f_0}{\partial c} + \frac{c^4}{\lambda} f_0\right) + \frac{1}{\lambda} v_x f_1 = 0 \tag{5}$$

If we form from the individual terms an average over the entire velocity space, we find that

$$\frac{1}{3}\frac{e_0}{m_e}(E_{d-c} + E_{hf} \cdot \cos\omega t)\frac{1}{c^2} \cdot \frac{\partial}{\partial c}\left(c^2 f_1 + c^2 g_1 \cos\omega t + c^2 g_2 \sin\omega t\right)$$

$$-\frac{m_e}{m_g}\frac{1}{c^2}\frac{\partial}{\partial c}\left(\frac{c^3 kT_g}{m_e\lambda_j}\frac{\partial f_0}{\partial c} + \frac{c^4}{\lambda} f_0\right) = 0 \qquad (6)$$

We multiply Eq. (5) by (v_x/c) and again form the averages over the entire velocity space. Again we have:

$$\omega(-g_1 \sin\omega t + g_2 \cos\omega t) + \frac{e_0}{m_e}(E_{d-c} + E_{hf} \cdot \cos\omega t)\frac{\partial f_0}{\partial c} + \frac{1}{\lambda} c f_1 = 0 \qquad (7)$$

We form time averages from Eqs. (6) and (7). Since apparently $E_{d-c} f_1 \gg \frac{\pi}{\omega} E_{hf} \cdot g_1$ (always when $\lambda \sim c$), equation (6) becomes

$$\frac{1}{c^2}\frac{\partial}{\partial c}\left(\frac{1}{3}\frac{e_0}{m_e} E_{d-c} c^2 f_1 - \frac{m_e}{m_g}\frac{kT_g}{m_e\lambda} c^3 \frac{\partial f_0}{\partial c} - \frac{m_e}{m_g}\frac{c^4}{\lambda} f_0\right) = 0 \qquad (8)$$

and similarly Eq. (7) becomes:

$$f_1 = -\frac{e_0}{m_e} E_{d-c}\frac{\lambda}{c}\frac{\partial f_0}{\partial c} \qquad (9)$$

Equations (8) and (9) enable us to calculate the functions f_o and f_1. They are proof of the fact that the high-frequency signal does not affect the plasma. We can also determine the functions g_1 and g_2. We can determine the function g_2 easily from Eq. (7). From this equation we form a time average between the limits $(\pi/2\omega)$ and $(3\pi/2\omega)$ Considering Eq. (9), we immediately obtain for g_2 the expression:

$$g_2 = -\frac{1}{\omega}\frac{e_0}{m_e} E_{hf}\frac{\partial f_0}{\partial c} \qquad (10)$$

Similarly, we calculate g_1 from Eq. (6). We multiply this equation by cos ωt and form the time average for ωt equal to zero and π . We find that

$$\left(E_{hf} \cdot f_1 \frac{\pi}{2} + E_{d-c} \cdot g_1 \frac{\pi}{2} + E_{hf} g_2 \frac{2}{3}\right) = \frac{\text{const}}{c^2}$$

Evidently we can assert that at any $\underline{c}$, i.e., when c = 0, the left-hand side of the latter equation will be finite. Then the constant will equal zero, of course. Since it is still true that $g_2 \ll f_1$, and g_1 can be taken absolutely (since g_1 represents the real component of the high-frequency specific conductivity), we find, ultimately:

$$g_1 = -\frac{e_0}{m_e}\frac{\lambda}{c} E_{hf}\frac{\partial f_0}{\partial c} \qquad (11)$$

As can be seen, g_1 is smaller than f_1 in precisely the proportion (E_{hf}/E_{d-c}). Thus it is true here also that $g_1 \ll f_1$.

From Eqs. (8) and (9) we can determine the actual basic distributive function f_o. If we insert f_1 from (9) into Eq. (8), we obtain a well-known

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expression (according to /1/), on the basis of considerations similar to those which led to Eq. (11), i.e.:

$$\frac{df_0}{f_0} = - \frac{c^3 \cdot dc}{\frac{kT_g}{m_e} c^2 + \frac{1}{3}\left(\frac{e_0 E_{d-c}}{m_e}\right)^2 \frac{m_g}{m_e} \lambda^2} \tag{12}$$

Equations (12), (9), (10), and (11) have already solved our entire problem. Knowing λ as a function of $\underline{c}$ is important in order to determine the other parameters of the plasma. However, this dependence of the mean free path on the speed of the electron can only be postulated outside Maxwell's distribution. For quasiisothermic plasma, however, the second term of the denominator of Eq. (12) loses its meaning, despite the calculated values /18,19,22,16,17/, so that this dependence does not affect the basic distributive function.

As we know, the current density $\underline{j}$ in the discharge is given by the expression $j = n_e e_0 \overline{v}_x$; $\overline{v}_x$ is the electron transport-velocity toward the field. For example, according to /1/, because of our equations (9) through (11), the current density is:

$$j = \frac{4\pi}{3} \frac{e_0^2 n_e}{m_e}\left[- E_{d-c}\int_0^\infty \frac{\partial f_0}{\partial c} \lambda c^2 dc - E_{hf}\left(\cos \omega t \int_0^\infty \frac{\partial f_0}{\partial c} \lambda c^2 dc + \sin \omega t \cdot \frac{1}{\omega}\int_0^\infty \frac{\partial f_0}{\partial c} c^3 dc\right)\right] \tag{13}$$

This equation indicates that total current density caused by a current of electrons is composed of d-c and high-frequency components. Thus if we denote the specific conductivity of the given environment by $\sigma = (j/E)$ we find that the specific conductivity of d-c current is given by:

$$\sigma_{d-c} = - \frac{4\pi}{3} \frac{e_0^2 n_e}{m_e} \int_0^\infty \frac{\partial f_0}{\partial c} \lambda c^2 dc \tag{14}$$

Similarly we obtain a relationship for high-frequency specific conductivity. This will, however, be divided into real and imaginary components; i.e., we obtain a complex specific conductivity. It follows from considerations such as those in /4/ and /5/ or even /6/ that the real component of the specific high-frequency conductivity σ_{real} is:

$$\sigma_{real} = - \frac{4\pi}{3} \frac{e_0^2 n_e}{m_e} \int_0^\infty \frac{\partial f_0}{\partial c} \lambda c^2 dc \tag{15}$$

Similarly, the imaginary component $(i = \sqrt{-1})$ of this conductivity σ_{imag} is:

$$\sigma_{imag} = -i \frac{4\pi}{3} \frac{e_0^2 n_e}{m_e \omega} \int_0^\infty \frac{\partial f_0}{\partial c} c^3 dc \tag{16}$$

If we further compare σ_{d-c} and σ_{real}, it is clear that:

$$\sigma_{d-c} = \sigma_{real} \tag{17}$$

which is important information. According to /5/ the total specific conductivity of the high-frequency signal in the discharge is given by the expression

$$\sigma_{hf\ total} = \sigma_{real} + \sigma_{imag} + i\omega\varepsilon_0$$

where ε_0 is the dielectric constant of a vacuum (in the MKS system). But as we shall see below, σ_{imag} is so great that ε_0 becomes negligible in comparison.

Equation (14) is well known, following from considerations of the mobility of electrons in plasma; but Eq. (15) and expecially Eq. (16), are new. Similarly (17) indicates a hitherto unknown relationship between the high-frequency signal and the d-c electric field in the discharge under our conditions.

3. MEASUREMENT AND ITS RESULTS

In evaluating Margenau's relationship for the specific complex conductivity of a high-frequency discharge, frequent use was made of resonance methods /7,25/. In the case of an arc, however, it is not possible to use them because of the resonance frequency of its plasma /18/. Nor was it possible, for reasons of cost, to use the coaxial system of /3/. Therefore, the classic method of three voltmeters was used, as in Fig. 1; the frequency used was 26.7 megacycles, while the comparison frequency was 14.2 megacycles.

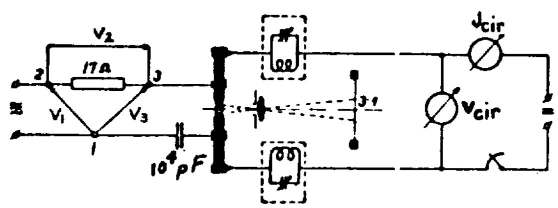

Fig. 1. Diagram of the method of measurement.

When $E_{hf} \lll E_{d-c}$ a value of one volt resulted for the output voltage of the oscillator. The oscillator was designed with an RV2P800 vacuum tube as an exciter, the output tube was an RD12RF, and amplitude regulation was provided by a 6J6 duodiode. The oscillator operated between 14 and 28 megacycles. Frequency was checked with an "Orion" wavemeter, while an Orion EMG 1321 electronic voltmeter was used for measuring high-frequency voltage.

We measured a d-c arc discharge occurring freely in the air between Noris Chromo carbon electrodes 11 mm in diameter. The arc was fed by a CKD rotating alternator: 2 kilowatts, 50 volts d-c.

No theory yet exists that even begins to explain the dependence of the diameter of the arc body on the supporting current. Thus it was necessary to measure the diameter of the arc body. Measurement was done in such a way that a picture of the arc was projected on a screen made of millimeter paper and enlarged exactly 3X times. Thus the length of the discharge was also determined precisely. Since the arc burned freely in air, it had the shape of a barrel, as can be seen in Figures 2 through 4. The maximum diameter of arc discharge burning in this fashion does not, of course, give an idea of the conduction of current, since the outer layers of the discharge body are primarily occupied by chemical reactions.

(For Figures 2 through 4 see Appendix II, page 368, not included in this translation).

A value lying between the maximum diameter and the diameter at the electrodes was taken as the effective diameter of the discharge body.

To prevent measurement in the high-frequency portion from being affected by electric-power equipment, parallel oscillating circuits were inserted between the arc discharge and the rotary alternator in each branch. The dynamic resistance resonance on the two branches at resonance was of the order of 3×10^4 ohms.

It is clear from the figure that the impedance of the arc is measured as a whole /3/. It is not possible to do otherwise, since no probe (which would of course be the most precise) can be used because of the high temperatures in the discharge. Radial probes containing carbon tips, which might be considered although they emit electrons, fail because of the F 2 4 undefined contact between the probe and the arc body. For the same reason, condenser plates cannot be used /20/. Measurement of the impedance of the arc as a whole is simple, although we are still faced here with ignorance of the precise value of the anode and cathode loss that occurs in the d-c supporting current, although not in the high-frequency signal.

In the measurement section of the apparatus, three voltages are read off between points 1,2, and 3. Voltage V_1 is the input voltage of the line, voltage V_3 is the voltage on the separating condenser plus the voltages on the arc discharge, output condensers, and resisters, while voltage V_2 is the voltage arising from the passage of a high-frequency current through 17 ohms of induction-free resistance.

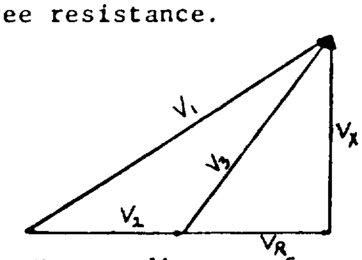

Fig. 5. Vector diagram of measured voltages.

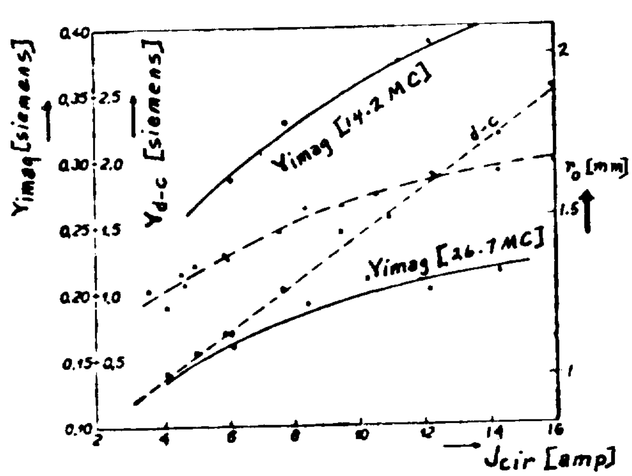

Fig. 6. Measured values of arc discharge when L = 0.5 cm.

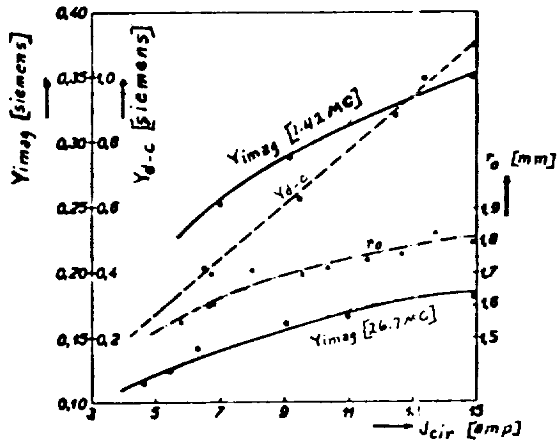

Fig. 7. Measured values of arc discharge when L = 1.0 cm.

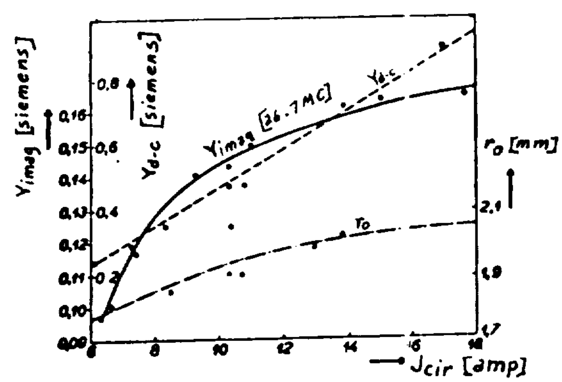

Fig. 8. Measured values of arc discharge when L = 1.5 cm.

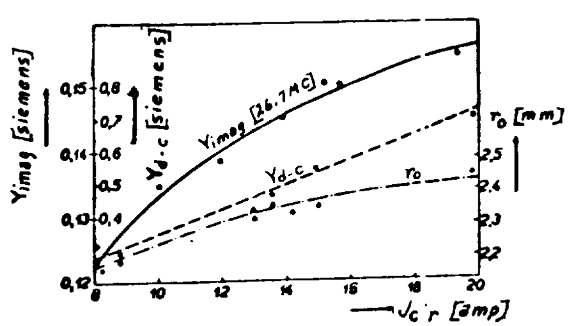

Fig. 9. Measured values of arc discharge when L = 2.0 cm.

According to Fig. 5, which shows the vector sum of the above voltages, we may write:

$$V_3^2 = V_x^2 + V_R^2 \qquad V_1^2 = V_x^2 + (V_2 + V_R)^2$$

so that:

$$V_R = \frac{V_1^2 - \left(V_2^2 + V_3^2\right)}{2V_2} \qquad V_x = \sqrt{V_3^2 - V_R^2} \tag{18}$$

If we denote the induction-free resistance of 17 ohms as R_Mk we obtain from Eqs. (18):

$$R = V_R \frac{R_M}{V_2} \qquad X = V_x \frac{R_M}{V_2} \tag{19}$$

If we thus measure the three voltages, i.e., V_1, V_2, and V_3, we can determine the total impedance:

$$Z = R - iX \tag{20}$$

where R and X are given by Eqs. (19). This impedance includes the impedance of the arc discharge, the separating condenser, and the output lines. If we know these added impedances, we can easily determine the total conductivity of the positive column of the arc.

The measurement section of the apparatus cannot be placed right next to the discharge, because of the temperature. This distance, which ranges from 5 to 8 centimeters, produces resonant oscillations of the high-frequency line, which in turn cause somewhat imprecise readings of voltage at certain discharge impedances. Therefore a calibration measurement was made on the basis of Eq. (17), which is actually confirmed experimentally in /3/. Assuming that the anode and cathode losses total 20 volts /3,16,17,18/, we can determine the resistance of the positive column from $V_{cir.}$, this is apparently decisive for the high-frequency signal, since this signal loses practically no energy in the anode and cathode portions of the discharge, because the dimensions of these portions correspond to the value of the mean free path in the given environment. On the other hand, when a d-c supporting current is used, both losses are a condition for the existence of the discharge. The calibrating impedance was developed from a parallel combination of ohmic resistance (corresponding to the d-c resistance of the positive column) and a capacitance. Agreement among voltages V_1, V_2, and V_3 was then found from changes in this capacitance. Then of course $Y_{d\text{-}c} = (1/R_{d\text{-}c})$ if $R_{d\text{-}c}$ is the ohmic resistance of the positive column, $Y_{d\text{-}c}$ its conductivity, and $Y_{imag} = \omega C$ the imaginary component of the conducticity of the positive column (C is the capacity used in the element). The calibrating element was placed between the burned ends of the electrodes such that even with considerable changes in the distance between electrodes the high-frequency voltmeter would show no changes in the measured values. Since the tip was about 2 centimeters from the working end of the electrode (Cf. Fig. 1), a change in the resistance of the carbon electrode had little effect on the calibration measurement.

If r_o is the effective diameter of the arc body and L the length of the body, the conductivity Y in terms of the specific conductivity σ is given by:

$$Y = \sigma \frac{\pi r_0^2}{L} \tag{21}$$

If Y_{d-c}, Y_{imag}, r_o, and L, are known from Figs.6 through 9, it is then possible to calculate σ_{d-c} and σ_{imag} for various lengths and supporting d-c currents in the arc discharge, as shown in Figs. 10 and 11.

In Eq. (16) the integral $\int_0^{\infty} (\partial f_0/\partial c)c^3$ is apparently independent of frequency. Thus, from (21) and (16):

$$\frac{Y_{imag}(\omega_1)}{Y_{imag}(\omega_2)} = \frac{\omega_2}{\omega_1} \tag{22}$$

Figures 6 and 7 give Y_{imag} for the frequencies of 26.7 and 14.2 megacycles. Here the frequency ratio is 1.88. If we take the value of Y_{imag} for 14.2 megacycles and divide it by the corresponding value (for the particular $J_{cir.}$ chosen of Y_{imag} for 26.7 megacycles, we obtain, (very approximately and always the same), a value, that varies within narrow limits around the average value for the two frequencies, as Eq. (22) requires.

The different values of σ_{d-c} and σ_{imag} for various discharge lengths L are explainable primarily in a freely burning arc. For various discharge lengths the shape of the body differs (cf. Fig. 2 through 4), and thus the cooling of the surface layers of the body is different. Naturally, therefore, there are different operating conditions for each discharge length, and thus the specific conductivities are all quite well defined.

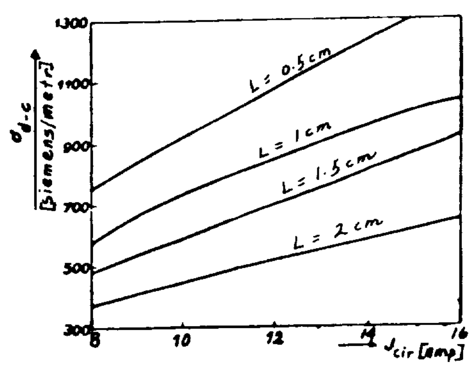

Fig. 10. Real component of specific conductivity.

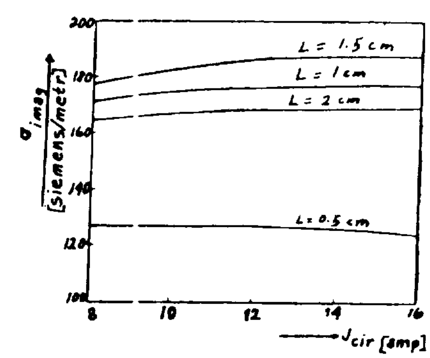

Fig. 11. Imaginary component of specific high-frequency conductivity (26.7 kilocycles).

4. DETERMINATION OF f_o; CALCULATION OF AVERAGE ELECTRON CONCENTRATION AND OF AVERAGE PLASMA TEMPERATURE

In any further computation the function f_o must be known. It can be determined from Eq. (12), which yields after integration:

$$f_0 = \text{const}\,\exp\left[-\int \frac{c^3 dc}{\frac{kT_g}{m_e}\,c^2 + \frac{1}{3}\left(\frac{e_0 E_{d-c}}{m_e}\right)^2 \frac{m_g}{m_e}\,\lambda^2}\right] \tag{23}$$

where konst. represents the integration constant. To calculate the exponent we must either know or choose the dependence of the mean free path λ on the electron velocity c. As Gvosdover /2/ first showed, and as follows from /1/, λ is generally dependent on velocity, owing to Coulomb forces. The degree of this dependence remains an open question. Thus, if we make λ proportional to various powers of the velocity c, we obtain various types of distributive functions, which more or less satisfy the particular operating conditions or type of discharge. For

$$\lambda = \frac{c}{\nu} \tag{24}$$

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we obtain a Maxwell type of distributive function, in which ν is the collision frequency of the electron, independent of c. Thus, if we substitute Eq. (24) in (23) and determine the integration constant /21/ according to kinetic theory, we finally obtain:

$$f_0 = \frac{\exp\left\{-\dfrac{c^2}{2\left[\dfrac{kT_g}{m_e} + \dfrac{1}{3}\left(\dfrac{e_0 E_{d-c}}{m_e \nu}\right)^2 \dfrac{m_g}{m_e}\right]}\right\}}{(2\pi)^{3/2}\left[\dfrac{kT_g}{m_e} + \dfrac{1}{3}\left(\dfrac{e_0 E_{d-c}}{m_e \nu}\right)^2 \dfrac{m_g}{m_e}\right]^{3/2}} \tag{25}$$

It is easy to determine $(\partial f_0/\partial c)$ from this equation. Then, however, Eq. (16) for the imaginary component of the specific high-frequency conductivity of the positive column yields:

$$\left|\sigma_{imag}\right| = \frac{e_0^2 \cdot n_e}{m_e \cdot \omega} \tag{26}$$

Similarly, from Eq. (14), if we substitute for $(\partial f_0/\partial c)$ and use Eq. (24), the specific conductivity of the d-c supporting current -- and thus actually σ_{real} according to (17) — for the positive column will be:

$$\sigma_{d-c} = \frac{e_0^2 \cdot n_e}{m_e \cdot \nu} \tag{27}$$

If we substitute the electron concentration from Eq. (26) in this equation the relation between the two specific conductivities will be given by:

$$\frac{\sigma_{d-c}}{\left|\sigma_{imag}\right|} = \frac{\omega}{\nu} \tag{28}$$

This completes the calculation of specific conductivities. When σ_{d-c}, σ_{imag}, and ω are known, it is easy, using Eqs. (26) through (28), to calculate the electron concentration n_e, the collision frequency ν, or the effective cross section of the electron, $\bar{Q}_e$.

When $\lambda = (c^n/\nu)$ and n is a common exponent, we obtain a Druyvestein distribution /9/ for $n = 0$ as $T_g \to 0$. Even with this distribution the equation for σ_{imag} comes out in conformity with Eq. (16), and similarly for n = (3/2). Of course the expressions for σ_{d-c} differ from Eq. (27). When $n > 2$, the kinetic equation does not yield a solution because of Davydov's term; in other words, the distributive function loses the sense of probability.

It is thus clear that Eq. (16) is somewhat broader in character, since it provides a rather simple form for σ_{imag}, given by Eq. (26). According to this equation the imaginary component of high-frequency conductivity is, with constant ω, dependent only on the electron concentration. Compared to the results of Margenau's theory /4/ this is a considerably simpler expression, since according to /4/ σ_{imag} is dependent on λ, while in addition the dependence on ω is directly proportional.

According to Fig. 11, when L = 1.5 cm, $J_{cir.}$ = 16A, and the frequency is 26.7 megacycles, σ_{imag} = 188 (siemens/meter). According to (26), however, this corresponds to an average electron concentration of:

$$n_e = 1.12 \times 10^{12} \text{ cm}^{-3}$$

This is a very good result if we consider that an arc discharge burning freely in the air was measured (i.e., a so-called Petrov arc).

It follows from Fig. 10 that when L = 1.5 cm and $J_{cir.}$ = 16 A, σ_{d-c} = 930 (siemens/meter). According to Eq. (28), therefore:

$$\nu = 3.39 \times 10^7 \text{ sec}^{-1}$$

It follows from Eq. (24) that the collision frequency ν is generally given by the product of the relative, or mean, electron velocity and the effective cross section multiplied by the appropriate number of particles. According to /1/ and /2/:

$$\nu = \overline{c}\left(n_g \overline{Q}_g + n_+ \overline{Q}_+\right) \tag{29}$$

where n_g is the concentration of neutral molecules or atoms, n_+ is the concentration of ions, $\overline{Q}_g$ is the effective cross section of electrons with respect to molecules, $\overline{c}$ is the mean velocity of electrons, and $\overline{Q}_+$ is the value of the effective cross section of the electron with respect to positive ions (first derived by Gvosdover /2/), $\overline{c}$ can be determined from Eq. (25). As for arc discharge at atmospheric pressure it is known /16,17/ that its plasma is very approximately isothermic and that the distributive function of electron velocities is Maxwellian. Then, of course, in Eq. (25) the term $\left[\frac{1}{3}\left(\frac{e_0 E_{d-c}}{m_e \cdot \nu}\right)^2 \frac{m_g}{m_e}\right]$ actually becomes negligible in comparison to the term (kT_g/m_e). We thus obtain a pure Maxwell distribution. For this distribution, however, the mean electron velocity is given by /21/ as:

$$\overline{c} = 2\sqrt{\frac{2}{\pi}}\sqrt{\frac{kT_g}{m_e}} \tag{30}$$

Furthermore, we may properly assume that n_e will equal n_+ /8,16,17,18,19/. If we choose $T_g \sim 5\times10^3$ deg. /16,18/ then, according to Eq. (30) and the previously calculated value of ν, it follows from (29) that $\left(n_g\overline{Q}_g + n_+\overline{Q}_+\right) = 0.767\ \mathrm{cm}^{-1}$. Compared to other types of discharge, arc discharge contains the greatest concentration of electrons and thus the greatest concentration of positive ions. Then, of course, the Coulomb forces will have the greatest effect on the electron path; _ will thus be primarily determined in Eq. (29) by the term $n_+\overline{Q}_+$. Moreover, $\overline{Q}_g$, determined from the concepts of kinetic theory, will not, in this case, apparently be as correct as Gvosdover's term $\overline{Q}_+$. Thus, if we eliminate the term $n_g\overline{Q}_g$, from Eq. (29), then it follows that $\overline{Q}_+ \cong 6.85 \times 10^{-13}\mathrm{cm}^2$ for the previously calculated value $n_e = n_+$. This value is very close to the value of $10^{-14}\ \mathrm{cm}^2$ given by Elenbaas /16/ and Brode /23,24/. Thus $\overline{Q}_+$ can be calculated approximately from measured values. Since, however, according to /2/:

$$\overline{Q}_+ = \frac{\pi}{2}\,2.303\left(\frac{e_0^2}{kT_g}\right)^2 \cdot \log\left(\frac{kT_g}{e_0^2 n_e^{1/3}}\right) \tag{31}$$

(using logs to the base 10), once we know $\overline{Q}_+$ we can approximately calculate the plasma temperature. From Eqs. (26, 27,30,31, and 29), eliminating the term $n_g\overline{Q}_g$, we find that:

$$T_g^{3/2}\Big/\log\left[\frac{k}{\left(e_0^4 m_e\right)^{1/3}} \cdot \frac{T}{\left(\omega \cdot \sigma_{imag}\right)^{1/3}}\right] \cong 2.303 \cdot \sqrt{2\pi e_0^2}\sqrt{\frac{m_e}{k^3}}\ \sigma_{d\text{-}c} \tag{32}$$

If we substitute here the values L = 1.5 cm, $J_{cir.} = 16$ A, $\sigma_{imag} = 188$ (siemens/meter) and $\sigma_{d\text{-}c} = 930$ (siemens/meter), we find that:

$$T_g \cong 6{,}580 \text{ degrees}$$

The result again corresponds to our previous knowledge of arc plasma /16,8/. Some values of n_e and T_g according to our measurements are given in Tables I and II. For comparison, concentration and temperature can also be determined from the values given in /3/. If we estimate $r_\rho \sim 0.5$ mm (in view of cooling of the discharge by linear air movement) we obtain, when $J_{cir.} = 4$A, the following values: L = 1 cm, $\sigma_{d\text{-}c} = 10$ (siemens/cm) and $\sigma_{imag} = 20$ (siemens/cm). At a frequency of 1,000 megacycles it turns out that $n_e \sim 10^{14}\mathrm{cm}^{-3}$, $T_g \sim 7{,}000$ degrees.

TABLE I.- L = 1.5cm

I_{cir}(A)	T_g(deg)	n_e(cm^{-3})
8	4,000	1.05×10^{12}
12	5,500	1.11×10^{12}
16	6,580	1.12×10^{12}

TABLE II . J_{cir} = 12A

L (cm)	T_g (deg)	n_e (cm^{-3})
0.5	7510	7.66×10^{11}
1.0	6200	1.055×10^{12}
1.5	5500	1.11×10^{12}
2.0	4280	1.01×10^{12}

F 2 4

As the results show, the notions of Maxwell's distribution and Gvosdover's effective cross section can be retained. If we consider all the terms in the kinetic equation, we arrive at a difficult integral-differential equation. That is why an energy balance is used for arc discharge, which of course only mean values of any of the plasma parameters. Thus, this method cannot be used. Vlasov has solved the kinetic equation in detail /26/, and most recently this was done by Yadavalli /27/, although both were done for special conditions.

It should be noted also that both Margenau's equation /4/ for complex conductivity and ours can be formally derived from the equation of motion for the electron by integrating it for $\omega \ll \nu$ on the assumption that all electrons have the same velocity. Of course this derivation is valueless for the physical foundation of complex conductivity.

5. CONCLUSION

The kinetic equation must provide a solution simultaneously for d-c current and for the high-frequency signal. It is thus clear that the overall distributive function must, according to Lorentz' approximation, be composed of cylindrical terms which would express the simultaneous passage of the d-c current and the high-frequency current through the plasma. This is satisfied by Eq. (3), which is thus the core of the solution.

Our measured values of $\sigma_{d\text{-}c}$ and σ_{imag} give very good results for the average electron concentrations and average plasma temperature according to existing knowledge. Similarly, calculation of n_e and T_g from the measured values in the work of Cobine, Cleary, and Gray supports the working procedure suggested here.

The conclusions reached will probably be usable in technical work for the rapid measurement of electron concentration in high-pressure discharges. From the theoretical standpoint, new laws have been discovered here concerning the passage of a weak high-frequency signal through a high-pressure plasma.

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